THE TWO GEOMETRIC CONCEPTS APPLIED ON THE ARCHITECTURAL DESIGN OF MEDIEVAL CHURCHES IN THE BALKANS

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Abstract: This paper presents a combined application of the two geometric concepts to analyze architectural layouts of selected medieval churches in the Balkans. One concept is a geometric construction, known as ***Štambuk's canon***, which is here supplemented by triangulation. Its application has been already attested to a wide range of late antique and medieval churches in the Balkans. The second concept is the *Octagon model*, which is based on the recovered geometric drawings incised in stone found at the Octagon monument, originally part of the late Roman Galerius Palace in Thessaloniki. Both these concepts rely on geometric shapes and specific type of grid, derived from the combination of basic shapes of circles with triangles and squares. In the first approach, the two types of triangular sequences, i.e. geometric patterns, are applied on the twelve layouts of the churches. The analysis shows that these patterns correspond with the four types of layouts, previously grouped by Nenadović. In the second approach, based on computer modeling and fitting the four churches' layouts into the complex Octagon model, the maximal deviation is +/−3 %. Although these are two diverse starting settings for geometric analysis, this study indicates a strong correlation between them. Namely, Štambuk's construction and Octagon model have been found to have several elements in common, such as the center of the apse, the edge of Štambuk's *holy triangle*.
This investigation additionally points to the possibility that the monuments from a wider Balkans region could have been designed by using similar basic geometric principles.
Keywords: geometric concept, late Roman architecture, medieval churches, Štambuk's canon, triangulature, Octagon model.

INTRODUCTION

It is a challenge for scholars to investigate the proportions and the underlying geometry of the diverse monuments built during medieval times, regardless of their stylistic characteristics and territorial locations. Considering the variety of such monuments, their age, and architectural features, currently, there is no scholarly consensus about reliable geometric data and proportional analysis applicable for the structures erected in the Balkans. Two recent publications highlight these ongoing questions about the design practices in the medieval Balkans and the Mediterranean. Referring to the work of Buchwald, who identified the quadrature grid at the tenth-century Myrelaion church in Constantinople (Istanbul),[1] Ousterhout points that:

"The dome module could form the basis for the entire plan and for the elevation, either through the application of simple geometric relationships or with a grid system, as is evident at the Myrelaion in Constantinople or at Sardis Church E. At the latter, foundations were laid in a grid, although the excavator suggests the plan may have been developed with a simple form of quadrature."[2]

Oikonomou asks whether the quadrature-based design applied for the triconch and tetraconch Byzantine churches, could be a theoretical design or a practical way of constructional tracing method.[3] Mamaloukos and Fountas additionally share the same idea concerning the ground plan drawing methods during the construction of Byzantine churches, which are mainly based on simple quadrature geometry and the use of ropes.[4]

This research joins this debate and investigates the underlying geometry of the group of the Christian Orthodox churches built in medieval Serbia, within the territories of Raška, sometime between the second half of the twelfth and the first half of the fourteenth centuries.[5] Since the nineteenth century, major attention of scholars has been on the structural aspects of these churches and stylistic characteristics of these monuments.[6] The specifics of the design patterns are often treated in a more descriptive manner, focusing on the utilization of the church spaces or the needs of liturgical services. Geometry of these churches, as an inseparable aspect of architectural design, is rarely analyzed, however.

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1 H. Buchwald. "The geometry of middle Byzantine churches and some possible implications", *JÖB* 42, 1992, 293–321.

2 R. Ousterhout. *Eastern Medieval Architecture: The Building Traditions of Byzantium and Neighboring Lands*, New York, 2019, 384.

3 A. Oikonomou. "The Use of Geometrical Tracing, Module and Proportions in Design and Construction, from Antiquity to the 18th Century", *International Journal of Architectural Heritage*, 2021, 8.

4 P. Fountas, Ο Ναός του Πρωτάτου, Διδακτορική διατριβή, Εθνικό Μετσόβιο Πολυτεχνείο, Αθήνα, 2008, 75–101; Σ. Μαμαλούκος [S. Mamaloukos], "Από τον σχεδιασμό στην κατασκευή: Ζητήματα εφαρμογής στη βυζαντινή αρχιτεκτονική / From Design to Construction: Aspects of Implementation in Byzantine Architecture," Δελτίον της Χριστιανικής Αρχαιολογικής Εταιρείας 4, no. 39, 2018, 83–97.

5 Gabriel Millet formulated the three major stylistic groups or architectural schools in medieval Serbia: the school of Raška, the Serbo-Byzantine school, and the Morava school. Gabriel Millet, *L'ancien art serbe: Les églises*, Paris, 1919. Because of the wide scholarly acceptance of such grouping of medieval churches in Serbia, his work remains predominant framework for the studies of the churches analyzed in this paper.

6 See, for example, J. Bogdanović, "Regional developments in late Byzantine architecture and the question of the "building schools". *Byzantinoslovica* LXIX, 2011. 1–2, 219–226, esp. 225.

Scholarly texts dealing with proportions and underlying geometry of medieval monuments usually apply one of the following three methodological approaches: finding the measurement units/group of units equal to a module for underlying dimensioning of the structure – (1); establishing geometric schemas: based on basic geometric shapes or grid – (2), and a combination of these two methods – (3). In this paper, method (2) is used above all because of incommensurability of contemporary and medieval units used in the first method.[7] Moreover, the Nemanjić family who sponsored building of the churches in Raška often brought craftsmen and masons form the Adriatic littoral, which inherited ancient Roman measuring system of units, yet the influence of Byzantine master builders (architects) and their system of measurements came through the cooperation of the church and political authorities.[8] Hence, Serbian scholars Bošković, Vulović, Čanak-Medić, and Vasiljević, among others, interpreted dimensioning of the separate spaces of the church and the whole by calculating and applying anthropomorphic measures used in medieval Raška.[9] This approach resulted in a wide range of measurement units from 30 to 45 cm. More unified conclusions are reached within the analysis of geometric schemes. Due to the structural logic of the church space, where the central core of the building is square-shaped (in the ground plan) transforming into a circle (at the level of the dome), scholars who applied geometric analysis, most often focused on a square, with its diagonals, repeatedly related to a circle (inscribed or circumscribed). Čanak-Medić, applied the square geometric schema as the underlying geometry of the churches St. Mary on the island of Mljet[10] and St. Nicholas in Toplica,[11] while Nešković[12] did the same for St. George in the monastery Đurđevi Stupovi. Čanak-Medić also combined *ad quadratum* (square-based) geometric schema with modular dimensioning in the analysis of the church of the Dormition of the Mother of God in the Studenica monastery.[13] Vasiljević applied the dynamic type[14] of the proportioning based on the triangular scheme, on the churches at the monasteries of Studenica and Dečani. Filipović introduced other geometric approaches based on

7 M. Zloković, „Antropomorfni sistemi mera u arhitekturi", In *Zbornik zaštite spomenika kulture IV–V*, Beograd, 1955, 181, noticed that "the measurements of a certain building based on contemporary units would be incorrect if that building was built based on another unit".

8 S. Vasiljević, „Naši stari graditelji i njihova stvaralačka kultura", In *Zbornik zaštite spomenika kulture*, 1952, 12.

9 Đ. Bošković and B.Vulović proposed a 30–31.4cm long measurement unit – *feet* – for the church of St. Nicholas in Kuršumlija, as well as a 45cm long measurement unit – *elbow* – for the katholikon in the Studenica monastery. In „Caričin grad – Kuršumlija – Studenica", *Starinar* VII–VIII, 1956–57, 177–178. M. Čanak-Medić and Đ. Bošković suggested that katholikon of the Studenica monastery was measured by *feet* 31.5cm long. In M. Čanak-Medić, and Đ. Bošković, *Spomenici Srpske srednjevekovne arhitekture. Korpus sakralnih građevina: Arhitektura Nemanjinog doba I*. Beograd. Republički zavod za zaštitu spomenika kulture Srbije. 1986, 97.

10 M. Čanak-Medić, *Spomenici Srpske srednjevekovne arhitekture. Korpus sakralnih građevina: Arhitektura Nemanjinog doba II*. Beograd. Republički zavod za zaštitu spomenika kulture Srbije, 1989, 158.

11 M. Čanak-Medić, „Postupci starih neimara pri projektovanju i izvođenju građevina". In *Razvoj nauke u oblasti građevinarstva i geodezije u Srbiji.* Referati sa simpozijuma održanog povodom 150 godina visokoškolske nastave u oblasti građevinarstva i geodezije. M. Sekulović i R. Mandić (ur.), 1996, 46.

12 Nešković, J. *Đurđevi stupovi u starom Rasu. Poreklo arhitekture crkve sv. Đorđa i nastanak Raških crkava u srpskoj srednjevekovnoj arhitekturi*, Kraljevo 1984, 106–108.

13 M. Čanak-Medić and Đ. Bošković 1986, op. cit. 97.

14 S. Vasiljević, op. cit., 18–24.

the visual effects, i.e. perspective of the observer entering the church, done in her analysis of St. George's church in the monastery Đurđevi Stupovi.[15]

To correlate the two general geometric principles – triangulature and complex grid in tracing the underlying geometry for the selected group of monuments, we tested the ground plans of medieval churches in the Balkans by applying the two distinct geometric concepts. One geometric concept, known as Štambuk's canon, is supplemented by *triangulature*. Štambuk devised a geometric scheme, where two circles and one equilateral triangle, which he called *holy triangle*, were combined in articulating the proportions of the church space. The applicability of the Štambuk's canon has been already attested to a wide range of late antique and medieval churches in the Balkans.[16] The second geometric concept is the Octagon model, which is attested in the geometric drawings from the late Roman Galerius Palace in Thessaloniki.[17] The Octagon model is based on a complex drawing, established by delicate geometry and principles of axial symmetry for eight radial directions, the squareness of a circle and double areas. It consists of eighteen geometric objects: four circles, eight squares and six intertwined and for 45° rotated squares each. Both these concepts rely on combinations of basic geometric shapes: circles, triangles, squares, and octagons.[18]

This research investigates the applicability of these two geometric concepts on the architectural designs of the selected group of eleven churches from the Raška territory and one church from the Adriatic littoral (used as an outlier).[19] Both concepts are investigated separately in authors' previous studies.[20] These two geometric concepts are tested on the ground plans of the churches built in the territory of medieval Raška state to determine whether there are common regularities and proportions in their architectural designs. In the process, the principles of *triangulature*, *quadrature*[21] and *octature*,[22] are expanded to the complex geometric schemes.

15 A. Filipović, „Hipoteza o projektovanju unutrašnjeg prostora crkve Đurđevi stupovi", *Starinar* LIX, 2009, 224–226.

16 I. Štambuk, „Zaboravljene proporcije. Kanon za projektovanje crkava", *Prilozi povijesti otoka Hvara* XI, pp. 91–109. Dragović et al., "Geometric proportional schemas of Serbia medieval Raška churches based on Štambuk's proportional canon", *Nexus Network Journal* 21(1), 2019, 33–58. Dragović et al. "Triangular proportional scheme and concept of the two Serbian medieval churches", in Proceedings of the 18th International Conference on Geometry and Graphics, ed. Luigi Cocchiarella, 40th Anniversary — Milan, Italy, August 3–7, 2018, 677–689. Dragović et al. "Geometric proportional model of the church of the Ljubostinja monastery", in Proceedings of the 1st International Conference SMARTART – Art and science applied, ed. Milan Prosen, Belgrade, Serbia, 2020, 423–434.

17 D. Savvides, "The conceptual plan of the octagon at Thessaloniki", *NNJ* 23(2), 2021, 401–405.

18 The shape of octagon is obtained by intertwining the squares which are rotated for 45°.

19 The church of St. Marija on the island of Mljet does not belong to the group of Raška monuments, but as commonly stated in the literature, it shares the same design principles of the dome structure with the katholicon of Studenica, as well as the global organization of the volumes in the exterior with the St. George's monastic church in Ras. In M. Čanak-Medić 1989, op. cit, 150 and 155.

20 Dragović et al., 2019, *eadem*; D. Savvides, 2021, *eadem*.

21 Bork highlights the rotation of polygons as an often used strategy in the proportioning of Gothic designs, and particularly points to the quadrature – rotation of a square as the best-known example. In R. Bork, "Ground plan geometries in Surger's St. Denis. A prototype for Altenburg", https://geometriesofcreation.lib.uiowa.edu/wp-content/uploads/sites/67/2020/06/BorkSDAltenberg.pdf

22 A star rosette at Burg cathedral (late Gothic monument) is inscribed in the sequence of rotated squares, hence obtaining the base octagon of the star. In D. Huylebrouck et al, "Octagonal geometry of the Cimborio in Burgos Cathedral", *NNJ* 13(1), 2011, (figures at) p. 200–201.

Mutual relations of the two geometric concepts are examined by comparing the geometric schemas and determining the elements they share.

The twelve monuments-churches, selected for this research due to their cultural and historical significance, architectural values and the diversity of shaping and contents, are listed in the Table 1. The two of them are at the UNESCO list of cultural heritage sites: the churches of the monasteries of Studenica (in 1986) and Sopoćani (in 1979).[23] Besides the variations in overall architectural form, the ground plans of these twelve churches differ in their architectural programs, presumably because some of the churches were built for a funerary purpose and additionally for different liturgical functions, some were monastic, others were court churches.[24]

Table 1: The list of the churches/monasteries, dates of their erection and founders' names

Church/location	Date of erection[25]	Founder[26]
ST. NICHOLAS near Kuršumlija	1158–1165	Stefan Nemanja
ĐURĐEVI STUPOVI	1167–1171	Stefan Nemanja
ST. MARY on the island of Mljet	1177	Benedictine authorities
STUDENICA near Ušće	1183–1191	Stefan Nemanja and his son
ŽIČA near Kraljevo	1208–1215	Stefan the First-Crowned
MILEŠEVA near Prijepolje	1234–1235	King Vladislav
PRIDVORICA near Ivanjica	1240/50	Unknown nobleman
MORAČA	1252	Stefan, Nemanja's grandson
SOPOĆANI near Novi Pazar	1255	King Uroš
GRADAC	1270	Queen Helen of Anjou
ARILJE	1295	King Dragutin
BANJSKA	1312–1318	King Milutin

METHODOLOGY

The two different geometric concepts are applied in the analysis of the ground plans of the twelve chosen churches to compare the two approaches and find the possible elements they have in common. One approach (conducted by the triangular geometry) accentuated the important points, spaces (entrance, center of the apse, i.e. altar table, altar), and dimensions (nave width, total exterior/interior width, etc.) of the church. The other (conducted by the complex geometry of Octagon model)

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23 https://whc.unesco.org/en/list/389 Studenica monastery; https://whc.unesco.org/en/list/96 Sopoćani monastery.

24 The katholicon of the Studenica monastery was built as a funerary church, while later it became a monastic church. For this purpose, king Radoslav, a grandson of Grand Ruler Stefan Nemanja (the founder of the church) also added the exonarthex with two side chapels. M. Čanak-Medić and Đ. Bošković, 1986, op. cit., 81.

25 The dates presented here are open to discussion, but they are not critical for the work presented. For example, M. Čanak-Medić dated the church of St. Nicholas in Toplice near Kuršumlija to the period 1158–1165, in M. Čanak-Medić, S*pomenici Srpske srednjevekovne arhitekture. Korpus sakralnih građevina: Arhitektura Nemanjinog doba II – Crkve u Polimlju i Primorju. Beograd, 1989, 150, while Đ.* Bošković dated the same church to the period 1168–1172 in Srednjevekovna arhitektura, Beograd, 1976, 282. Without going into debate about dating of some of the here analyzed churches, here we follow M. Čanak-Medić.

26 The question of patronage of some of the here analyzed churches has been also contested in recent scholarship. Again, without going into debate about the patronage of some of the analyzed churches, here we follow M. Čanak-Medić.

is linked to the structural logic of the church, starting from the dome structure and developing the design, through the geometry, towards final patterns, i.e. layouts.

The first geometric concept is based on the Štambuk's canon enriched by additional triangulation.[27] It is related to the more general dimensions of the church: width of the nave, total length/width of the church, and the dimensions of the altar space. The applied geometric schema slightly varies due to the diverse architectural outlines of the churches and their dimensions. The center of the apse, the most important point in the church from the liturgical point of view, remains a fixed geometric point throughout the analysis. By inscribing a sequence of equilateral triangles, starting with Štambuk's *holy triangle*, in the church nave, we obtain the parameter (edge of the triangle) for the length of the church, while additional triangles parametrize dimensions of the altar space and the total width of the church.

The other concept relies on the geometry of a complex grid named Octagon model – a specific set of shapes (squares and circles) in a complex composition (their geometric and algebraic relations are elaborated by Savvides).[28] The ground plans of the churches are scaled aiming to fit into the Octagon model while preserving the existing proportions of the monuments. Afterwards, it is examined whether other architectural elements of the churches could be derived from the Octagon model. Then, the deviation between the dimensions of the monuments and the corresponding model are calculated and evaluated.

This study, as originally proposed by Nenadović, confirms the necessity of presenting several types of architectural layouts for the purpose of further geometric analyses of the church ground plans. In his schematic representation of the ground plans for the Raška architectural group of churches, Nenadović formulated four different schemas.[29] These are: the basic – central space of the church, *naos* (three aisles along the longitudinal axes where the central one is under the dome, and the one on the East is in the altar) is upgraded with small vestibules – ***1***; choir spaces are in the form of the fake transept, while the side chapels are attached to the *narthex* – ***2***; the *pastophoria* are set as separate spaces beside the altar – ***3***[30]; the two towers are set in front of the narthex while elongated chapels are attached to the nave – ***4*** (Fig. 1). However, there are some deviations from these schemas among the selected ground plans of the churches. The length of the three aisles of the *naos* varied and sometimes even one of the aisles is missing. At the church in the Mileševa monastery, the third-east aisle is missing, while in the church of St. George in Ras, the central aisle under the dome is directly connected to the narthex, without the west aisle.[31] Table 2 presents the spatial organization of the examined churches in terms of their schematic representations, i.e. presence of: three aisles

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27 Dragović at al., 2019, op. cit.

28 D. Savvides, 2021, op. cit. 405–406.

29 S. Nenadović, *Arhitektura u Jugoslaviji od VIII–XVIII veka*, Beograd, 1987, 31.

30 In her analysis of the churches being built around 1300 M. Čanak-Medić finds that these monuments followed the architectural design that was previously applied in the Žiča monastery. Čanak-Medić, M. *Arhitektura Sv. Ahilija i raško crkveno graditeljstvo oko 1300 godine*, In Sveti Ahilije u Arilju. Istorija, umetnost. Zbornik radova sa naučnog skupa, Zavod za zaštitu spomenika kulture, 1996, 35.

31 One of the methods of establishing the dimensions of the aisles is based on the diameter of the dome. M. Čanak-Medić used this method in the discussion on the structural concept, and the dimensioning of several Raška churches: in the monasteries of Mileševa, Morača, Sopoćani and Arilje, M. Čanak-Medić, „Arhitektura Sv. Ahilija i raško crkveno graditeljstvo oko 1300 godine", in *Sveti Ahilije u Arilju: istorija, umetnost*, Zbornik radova sa naučnog skupa održanog 25–28 05. 1996, 36–37.

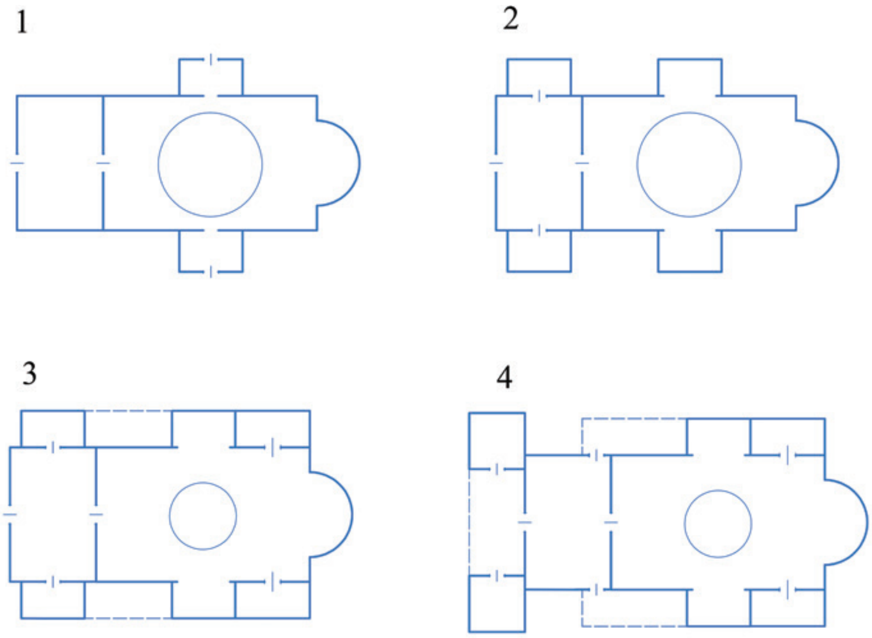


Fig. 1

(east, under the dome and west) in the *naos*; chapels attached to the narthex/exonarthex; and presence of towers. Because the organization of separate spaces for varying schemas of the selected churches strongly influences their dimensions and subsequently their underlying geometries, this type of analysis will contribute to the final conclusions regarding the relations of architectural schemas and adequate underlying geometries. [32]

Table 2 The overview of the architectural programs for the analyzed churches

CHURCHES / SPACE		St. Nicholas	Đurđevi stupovi	Studenica	St. Maria	Žiča	Mileševa	Pridvorica	Morača	Sopoćani	Gradac	Arilje	Banjska
Altar/p		3A	3A	3A	A/p	A/p	A/p*	A/p	A/p	A/p	3A	A/p	A/p
NAOS	East	+	+	+	+	+	–	+	+	+	+	+	+
	Dome	+	+	+	+	+	+	+	+	+	+	+	+
	West	+	–	+	+	+	+	+	+	+	+	+	+
Narthex/ch			+/2t	+	–	+/ch	+	+	+/1ch	+/ch	+/ch	+	+/ch
Exonarthex[33]		+/2t		+/ch			+/ch					+	+/2t
SCHEMA		2	2	1	1a	3	3	3	3	3	2	3	4

Several abbreviations are used in Table 2: 3A – tripartite altar space; A/p – altar space with separate *pastophorie*; ch – chapels; 2t – two towers.

32 Most of the elaborated churches have exonarthexes added in the construction phases later than the original church structure, while sometimes one or two towers were attached to the exonarthex.

GEOMETRIC ANALYSES OF LAYOUTS OF THE CHURCHES BASED ON ŠTAMBUK'S CANON AND ADDITIONAL TRIANGULATION

Geometric analysis of several churches from the Raška architectural group started with the premise that the *holy triangle* – an equilateral triangle (***ABC***), with edge equal to the width of the nave (***AB***=***n***), and related to the center **C** of the apse, could be the key element for designing and dimensioning of these churches.[33] Starting with this proposition, the elements of Štambuk's construction are set over the ground plans of the twelve selected churches.

Due to the diversity of shaping and dimensioning of the churches and their interior spaces, in this investigation some elements of Štambuk's canon were adjusted to the patterns of the architectural ground plans. Hence, circle $\boldsymbol{k_1}$ has three variations: coincident with the exterior edge of the apse – ***type 1***; concentric with exterior/interior edge of the apse – ***type 2***; coincident with interior edge of the apse – ***type 3***. In the following Table 3, all the related parameters of the proposed geometric setting are given in metrical units, for all the examined ground plans of churches.

Table 3 Overview of dimensions of geometric elements – Štambuk's canon and additional triangulation

Type	Church	r_1 (m)	r_2 (m)	ra_{ext} (m)	ra_{int} (m)	n_{ext} (m)	n_{int} (m)	T_{ext} (m)	T_{int} (m)	w_a	L (m)
2	ST. NICHOLAS	1.68	2.33	1.68	0.78	7.18	–	13.93	–	4.67	12.47
2	ST. GEORGE	2.15	2.89	5.79	1.45	8.81	–	16.62	–	5.46	15.58
1	ST. MARIA	2.24	3.13	2.24	1.49	8.62	–	–	–	6.08	14.94
2a	STUDENICA	2.99	4.09	2.98	1.96	9.87	7.82	14.89	–	6.28	24.09
1	ŽIČA	2.99	4.15	3.47	2.76	8.30	8.12	14.67	13.29	6.53	21.22
3	MILEŠEVA	3.28	4.48	4.13	3.28	9.21	–	–	11.23	4.74	–
3	PRIDVORICA	2.63	3.62	3.38	2.63	7.22	–	12.08	–	5.72	–
1	MORAČA	3.01	4.11	3.01	2.14	8.11	8.02	14.02	12.92	6.25	21.05
1	SOPOĆANI	3.12	4.25	3.97	2.67	8.41	6.21	13.71	12.90	6.28	22.13
2a	GRADAC	2.63	3.64	2.63	1.69	9.50	7.19	14.08	–	7.05	22.68
3a	ARILJE	2.49	3.39	2.98	2.03	7.04	5.05	12.38	–	5.20	16.14
3a	BANJSKA	3.23	4.40	4.29	3.23	8.81	7.21	15.08	–	–	27.47

Table legend: r_1 – radius of circle k_1; r_2 – radius of circle k_2; ra_{ext} – exterior radius of the apse; ra_{int} – interior radius of the apse; n_{ext} – exterior nave width; n_{int} – interior nave width; L – triangulation sequence length (distance from the entrance wall to the center of the apse – C); T_{ext} – total exterior width of the church; T_{int} – total exterior width of the church; w_a – altar interior width.

If one sets the value of $\boldsymbol{r_1}$ (a smaller circle – attribute of the main apse), then $\boldsymbol{r_2}$ and ***n*** are also defined, because of their strict constraints in Štambuk's construction. It is proposed that ***AB=n***, the edge of the *holy triangle* is equal to the nave width (exterior or interior) or a dome diameter ***AB=d***. Such modification of initial Štambuk's rule enables us to cover the diversity of the ground plans for the churches examined with triangular underlying geometry. Beside **Štambuk's** geometric construction, at the ground plan of each church, additional equilateral triangles are incorporated with the main schema. Their role is to reveal the other regularities present in the

33 This approach is elaborated in detail in Dragović et. al, 2019, op. cit., 39–43.

ground plan of a particular church, e.g., width of the altar space, position of the side chapels, or total width of the church.

The key point for Štambuk's construction is center of the apse **C**, while we adjusted $\mathbf{r_1}$ to be one of the above-mentioned possible options (the three variations of the circle $\boldsymbol{k_1}$). The edge of a *holy triangle* ABC is constrained with $\boldsymbol{r_1}$, and if necessary, additionally extended towards the exterior width of the nave, in order to obtain the sequence of triangles which determine dimension ***L*** – the distance from the entrance wall to the center ***C*** of the main apse. The relevant dimension of the church – total width, which depends on the depth of the lateral spaces of the church (e.g. choir or chapels) – is related to the characteristic mid points on the west entrance wall and/or the top point of the apse on the east, by setting the two "large" equilateral triangles. In a similar way, the dimensions of the altar space (particularly, its depth) are defined by smaller equilateral triangles, related to the top points of the main apse. Graphic analyses of each examined church are given in group of figures Fig. 2 a–c. Detailed elaboration of the characteristic parameters for the underlying geometry of these graphics are given in Table 4.

The results of the applied geometry show several regularities:

– one equilateral triangle – *holy triangle* from Štambuk' s construction, related to the center of the apse by its top vertex, is the key geometric element which determines the width of the church nave.

– the two identical equilateral triangles, joined by a common edge, and related to the center of the apse, approximate the dimensions of the naos, where the ratio width/length is $\mathbf{n:n\sqrt{3}}$ (this regularity is shared by all the twelve churches);

– similarly, the rectangle 1234 framing of the three aisles of the nave (east/altar, middle/dome and west) shares the same proportion $\mathbf{n:n\sqrt{3}}$;

– the sequence of the three triangles along the longitudinal axis approximates the distance from the main entrance of the church to the center of the apse, i.e. the length of the church without the apse (this regularity is shared by seven churches);

– less monumental churches and the ones which do not have all the three aisles, share the approximated dimensions obtained only by two equilateral triangles (this regularity is shared by five churches);

– in addition, the characteristic "large" equilateral triangles, related to the midpoint of the entrance wall (on the west side of the church), and/or to the top point of the apse (on the east side of the church), define the total exterior or interior width of the church;

– one smaller triangle, related to the top point of the apse defines the width of the altar space.

GEOMETRIC ANALYSES OF THE LAYOUTS OF THE CHURCHES BASED ON THE OCTAGON DIAGRAM

The second geometric analyses approach aims to quantify the applicability of a particular grid as the underlaying geometry of the ground plans of the four chosen monuments. This is a smaller sample of churches, which share better construction performances (orthogonality of the walls and symmetry) and simultaneously cover a diversity of designs (from simpler to complex ones). The two phases in this part of investigation were formulated: testing the principles of the Octagon model, based on the *octagon diagram* – 1, and determining the common elements of the Octagon model and Štambuk's canon with additional triangulation – 2. These principles are

Table 4a – Characteristic parameters of the underlying geometry: churches 1-4

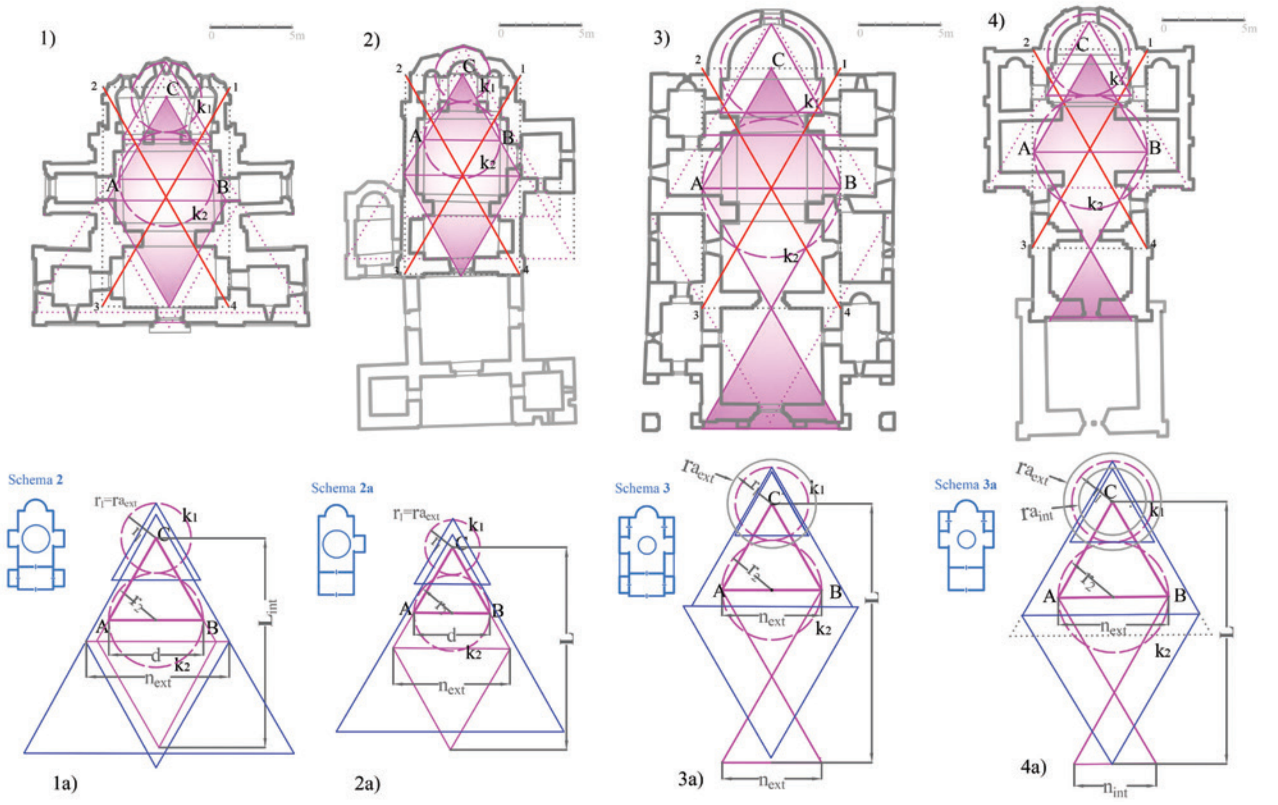
	Churches	Đurđevi stupovi	St. Nicholas	Arilje	Sopoćani
	Scheme/concept	**2**/1	**2a**/1	**3a**/2	**3**/2
Štambuk canon	k_1- circle	circumscribed APSE $r_1=ra_{ext}$	circumscribed APSE $r_1=ra_{ext}$	concentric APSE	concentric APSE
	k_2- circle	inscribed DOME (D_d)	inscribed DOME (D_d)	inscribed NAVE ($d=n_{ext}$)	inscribed NAVE ($d=n_{ext}$)
	"holy" triangle	edge - D_d	edge - D_d	edge - n_{ext}	edge – n_{ext}
	small triangle	edge- $D_{d,}$ ALTAR int. width	edge- $D_{d,}$ ALTAR int. width	edge- n_{int} ALTAR	edge- n_{int} ALTAR
Triangles	Δ sequence vertex C	2 identical, $2h= L_{int}$	2 identical, edge- n_{ext} ; $2h=L_{int}$	2 identical+1 $2h_1+h_2=L_{ext}$	3 identical $3h= L_{ext}$
	Δ large	East edge – W_{Text}	East edge - W_{Text}	East +West edge - W_{Tint}	East +West Edges -W_{Tint}; W_{Text}
	Δadditional	edge - n_{ext}	-	-	-

Table 4b – Characteristic parameters of the underlying geometry: churches 5-8

	Churches	Gradac	Studenica	Žiča	Morača
	Scheme/ concept	**2**/1	**1**/1	**3a**/2	**3**/1
Štambuk canon	k_1- circle	circumscribed APSE; $r_1=ra_{ext}$	circumscribed APSE; $r_1=ra_{ext}$	concentric APSE	circumscribed APSE; $r_1=ra_{ext}$
	k_2- circle	inscribed NAVE; edge-n_{int}	inscribed NAVE; edge-n_{int}	inscribed NAVE; edge- n_{ext}	inscribed NAVE; edge- n_{ext}
	"holy" triangle	edge - n_{ext}	edge - n_{ext}	edge - n_{ext}	edge - n_{ext}
	small triangle	edge- n_{int} ALTAR	edge- C_{arch} ALTAR	edge- n_{int} ALTAR	edge- n_{int} ALTAR
Triangles	Δ sequence vertex C	2 identical+1 $2h_1+h_2=L_{ext}$	2 identical+1 $2h_1+h_2=L_{ext}$	3 identical $3h= L_{ext}$	3 identical $3h= L_{ext}$
	Δ large	East +West edge - W_{Text}	West edge - W_{Text}	East +West Edges -W_{Text}; W_{Tint}	East +West Edges -W_{Tint}; W_{Text}

Table 4c – Characteristic parameters of the underlying geometry: churches 9-12

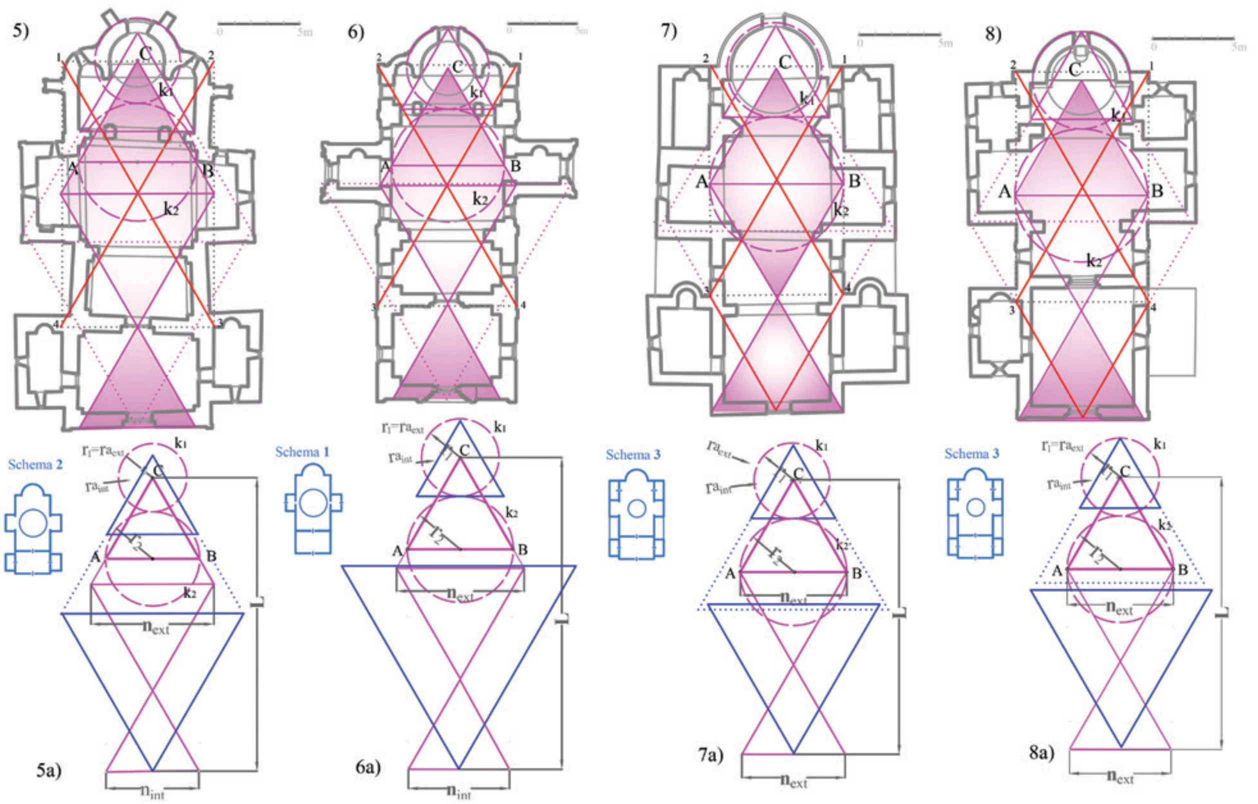
	Churches	Mileševa	Pridvorica	Banjska	Sv. Marija
	Scheme/ concept	**3a**/3	**3**/3	**4**/3	**0**/1
Štambuk canon	k_1- circle	inscribed APSE; $r_1=ra_{int}$	inscribed APSE; $r_1=ra_{int}$	inscribed APSE; $r_1=ra_{int}$	circumscribed APSE; $r_1=ra_{ext}$
	k_2- circle	inscribed NAVE; edge- n_{ext}	inscribed NAVE; edge- n_{ext}	inscribed NAVE; edge- n_{ext}	inscribed DOME (D_d)
	"holy" triangle	edge - n_{ext}	edge - n_{ext}	edge - n_{ext}	edge - D_d
	small triangle	edge- A_{arch} ALTAR	edge- n_{int} ALTAR	edge- Wa_{int} ALTAR	edge- C_{arch} ALTAR
Triangles	Δ sequence vertex C	2identical (+1) edge- n_{ext} ; $2h=L_{ext}$	2 identical edge- n_{ext} ; $2h=L_{ext}$	2+2 both identical;	2identical edge- n_{ext} ; $2h=L_{int}$
	Δ large	East, West edge – W_{Tint}	East edge – W_{Text}	East edge – W_{Text}	2identical edge- n_{ext}



1)
2)
3)
4)
0
5m
A
B
C
k1
k2
Schema 2
Schema 2a
Schema 3
Schema 3a
n ext
n int
L int
d
1a)
2a)
3a)
4a)

Fig. 2a

Fig. 2b



5)
6)
7)
8)
0
5m
A
B
C
k1
k2
Schema 2
Schema 1
Schema 3
n ext
n int
5a)
6a)
7a)
8a)

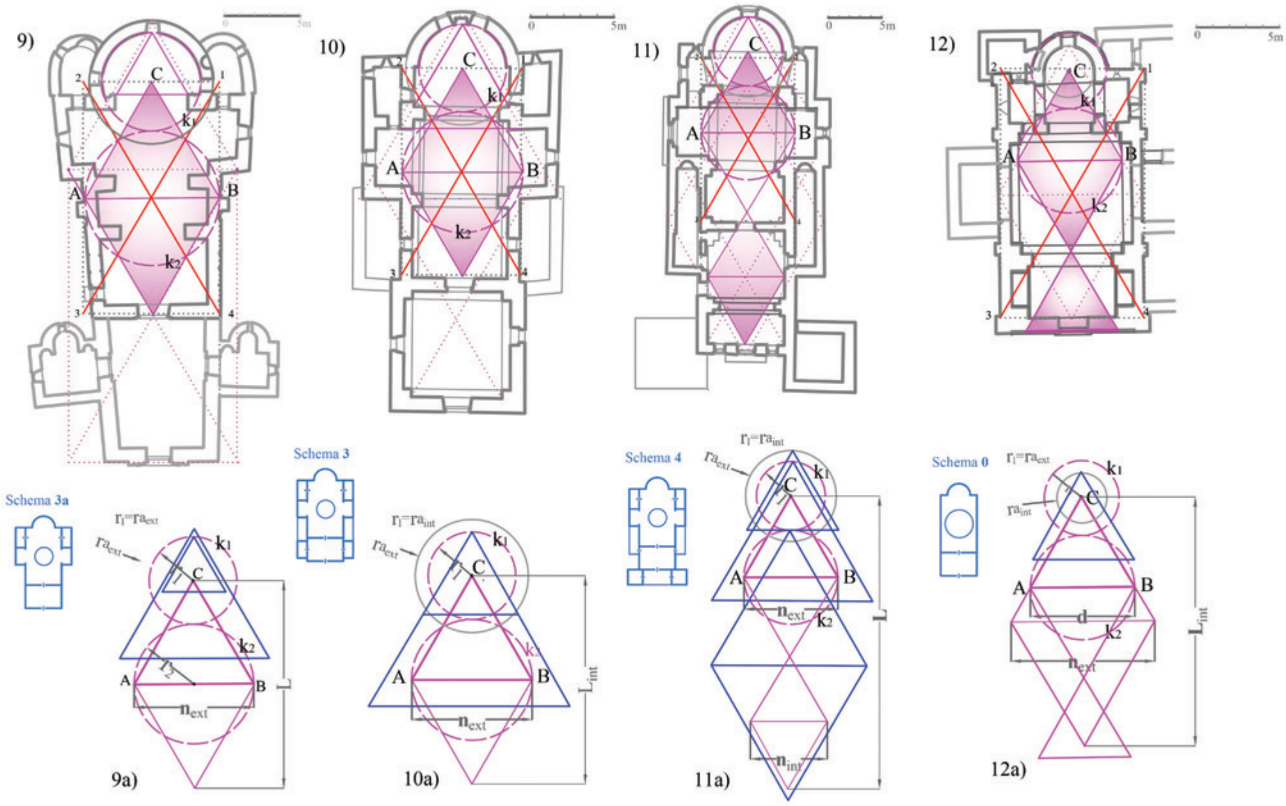


Fig. 2c

applied over the four monuments of ecclesiastical architecture: Arilje, Sopoćani and Studenica churches, along with the church of St. Mary on the island of Mljet.

Savvides suggested that the complex incisions on the stone in the Galerius Palace could provide the evidence concerning the implication of conceptual designs in late Roman architecture.[34] The drawing consists of 18 geometric objects: circles, squares and intertwined squares, which intensively form sequences of octagons. The complexity of the Octagon model presents a set of algebraic and geometrical sequences that describes both the basic geometric elements of the complex drawing and most of the architectural elements of the ground plan of the Octagon monument itself.[35] Mutual relations of geometrical objects incised on the stone beam are based on geometric concepts such as the *ad quadratum*, *circle squaring* and *octature*.[36] The algebraical values of the elements of the Octagon model, i.e. the set of geometric shapes: circles (I, II, IV and V) and squares (III, VI, VII and VIII) in order of appearance, successively from the center (origin of the coordinate system 0,0) of the model.

34 D. Savvides, 2021, op. cit., p. 422.

35 The building whose shape attested the purpose of the incisions found in Gallerius complex in Thessaloniki is named Octagon.

36 “While a circle circumscribed around a square by *quadrature* has a diameter 1.414 times as great as the square’s side length, the *octature* operation gives a circle with diameter 1.082 times the octagon’s width. In R. Bork, “Dynamic unfolding and the conventions of procedure: Geometric proportioning strategies in Gothic architectural design”, Reprint from *Proportional Systems in History of Architecture*, Leiden University Press, 2018, p.320

Table 5 Characteristics of the elements of the Octagon model, where the constant $\alpha=\sqrt{(2+\sqrt{2})}/2$.

Octagon model	∅ +	∅	I	II	–	III	+	–	IV	V
values	$\alpha\sqrt{2}$	$\sqrt{2}$	2	$\sqrt{2\pi}$	$\alpha\pi$	π	$\frac{\pi}{\alpha}$	$\alpha\pi\sqrt{2}$	$\pi\sqrt{2}$	$\pi\sqrt{\pi}$

Octagon model	–	VI	+	–	VII	+	VIII	+
values	$\frac{\alpha\pi^2\sqrt{2}}{2}$	$\frac{\pi^2\sqrt{2}}{2}$	$\frac{\pi^2\sqrt{2}}{2\alpha}$	$\alpha\pi^2$	π^2	$\frac{\pi^2}{\alpha}$	$\pi^2\sqrt{2}$	$\frac{\pi^2\sqrt{2}}{\alpha}$

The elements of the Octagon model (Table 5) are divided into three groups:[37]

Group A includes eight geometric objects in which equality between areas of circles and squares is identified: The ratio between the radiuses of circles and the sides of squares III/I and VI/IV is equal to $\pi/2$. Hence, the squaring areas of circles I and IV are equal to the areas of squares III and VI respectively; the ratio between the radiuses of circles II–I and V–IV is equal to $\sqrt{2}/2$. This relation means that each one of the squares that has an area equal to the area of circles I and IV is inscribed by the circles II and V; moreover, the arithmetical ratio between the sides of the squares corresponding to IV/III, VII/VI and VIII/VII is equal to $\sqrt{2}$. Therefore, the area of square VI is the half of VII which is the half of VIII.

Group B includes the three squares constructed by rotating of the primary squares for 45°, related to positions III, VI and VII.

Group C includes seven squares which are attached to every previously described square. The ratio between the sides of these squares and their attached squares is calculated either to α (squares III and V and their corresponding rotated squares, denoted with the symbol **–**) or 1/α (squares VII and VIII and the rotated square of VII; denoted with the symbol +).

Geometric analysis of the four ground plans based on the Octagon model starts with the premise that the ring of the central dome coincides with the basic elements of the Octagon model-circle or the square (eg. circle I is applied at Sopoćani and Arilje, while square III is applied at Studenica and St. Mary). Considering that the radius of *circle I* is equal to the value of 1 (non-dimensional), the scaling of the Octagon model is performed in the following scales: for Studenica 1:267, Sopoćani 1:211, Arilje 1:154.9 and St. Mary 1:178.55. In order to calculate the deviations of the model from the ground plans a digitization process is applied. Graphic interpretation of this process, which is performed in commercial software Digitizer 2.6.8, is presented on the example of the St. Achillius church in Arilje (Fig. 3).

The red squares in Fig. 3 represent the digitized points for the church ground plan that are close to the Octagon model, while the red dots stand for the points of the Octagon model that are suggested to predict the coordinates of the church ground plan. The embedded figure, at the top right of Figure 3, exhibits the deviation between the model and the actual ground plan for each one of the above coordinates. When the red dot is in the middle of the red square the error is 0, while when the dot is at the corner of the square the error is 3 %.

37 D. Savvides, 2021, op. cit., 407.

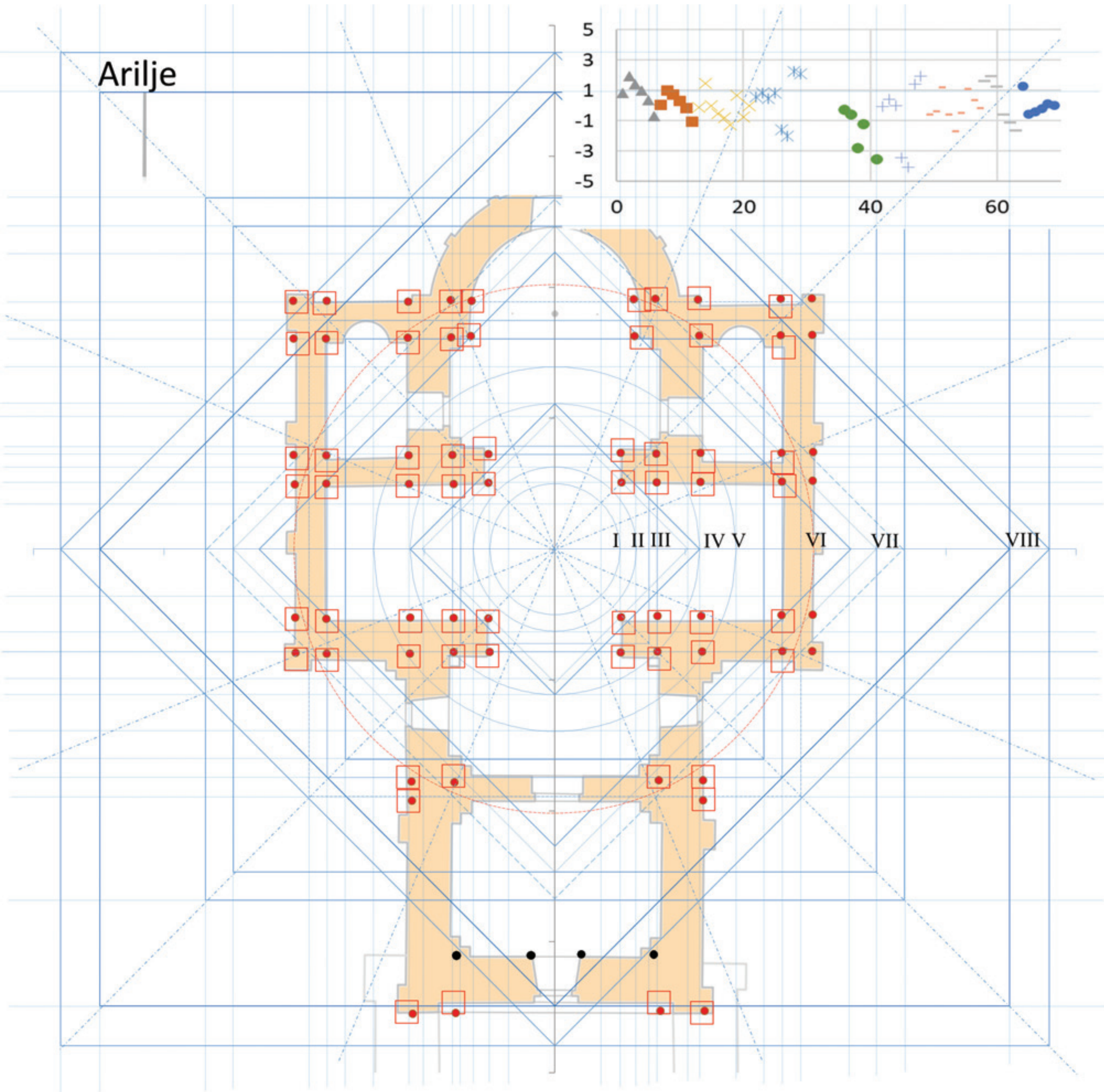


Fig. 3

The same procedure is repeated for the three other churches. Results show that the Octagon model can satisfactorily define most of the elements of the ground plans of the examined churches (such as side walls, radius of domes etc.) with an overall deviation of less than ±3.0 %.

The final results, after the validation of the Octagon model for the four chosen churches, are presented in Fig. 4 a–d. Architectural elements of the examined churches, that are successfully described (deviation of less than ±3 %) by the Octagon model, are underlined with red thick lines. The red arrows indicate the elements of the Octagon model that present the modeled lines of the ground plans.

The correlations between the elements of the Octagon model and the real dimensions of the ground plans of the four churches are presented in diagrams in Fig. 5 a–d. Both data (modeled and measured) are on a very strong linear correlation (R^2~1), which suggests that the Octagon model can predict the most parts of the examined church layouts.

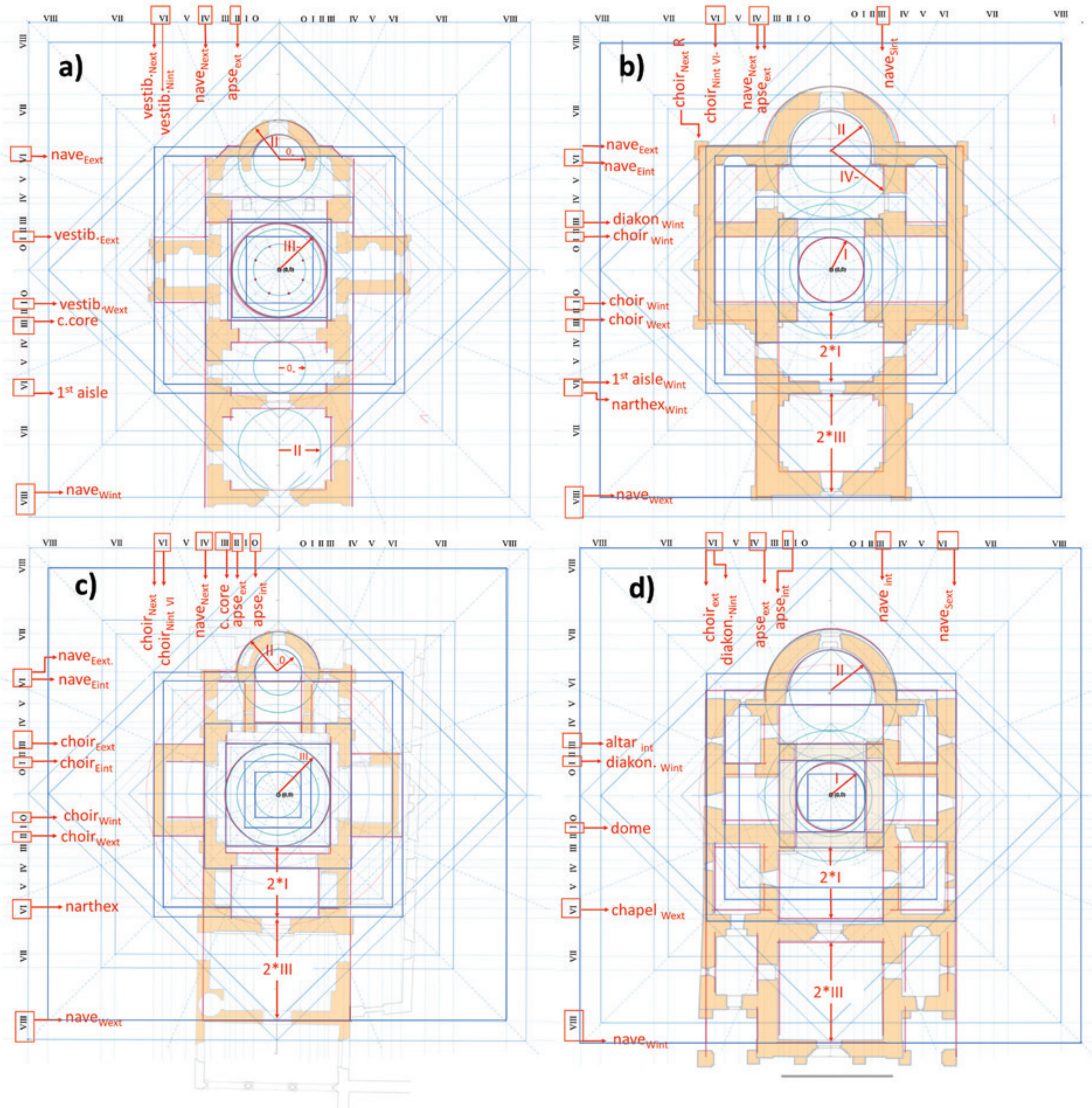

a)
vestib.Next
vestib.Nint
naveNext
apseext
naveEext
vestib.Eext
vestib.Wext
c.core
1st aisle
naveWint
b)
choirNext R
choirNint VI-
naveNext
apseext
naveSint
naveEext
naveEint
diakon Wint
choir Wint
choir Wint
choir Wext
1st aisleWint
narthexWint
naveWext
2*I
2*III
c)
choirNext
choirNint VI
naveNext
c. core
apseext
apseint
naveEext.
naveEint
choirEext
choirEint
choirWint
choirWext
narthex
naveWext
2*I
2*III
d)
choirext
diakon.Nint
apseext
apseint
nave int
naveSext
altar int
diakon. Wint
dome
chapel Wext
naveWint
2*I
2*III

Fig. 4

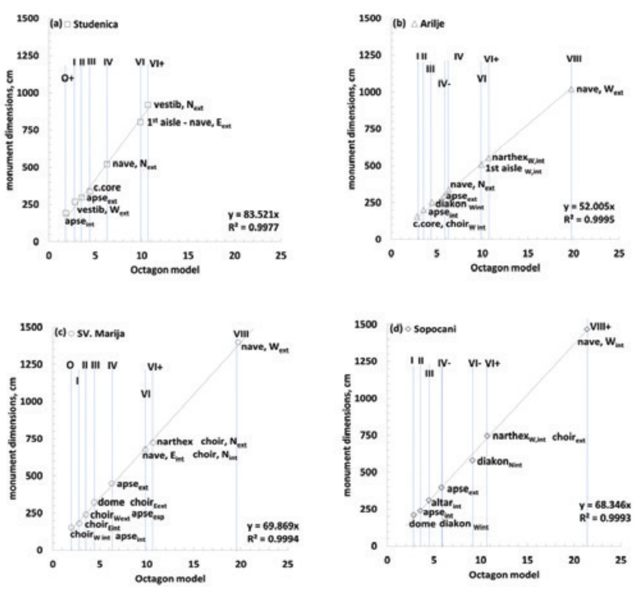

(a) □ Studenica
y = 83.521x
R² = 0.9977
(b) △ Arilje
y = 52.005x
R² = 0.9995
(c) ○ SV. Marija
y = 69.869x
R² = 0.9994
(d) ◇ Sopocani
y = 68.346x
R² = 0.9993
monument dimensions, cm
Octagon model

Fig. 5

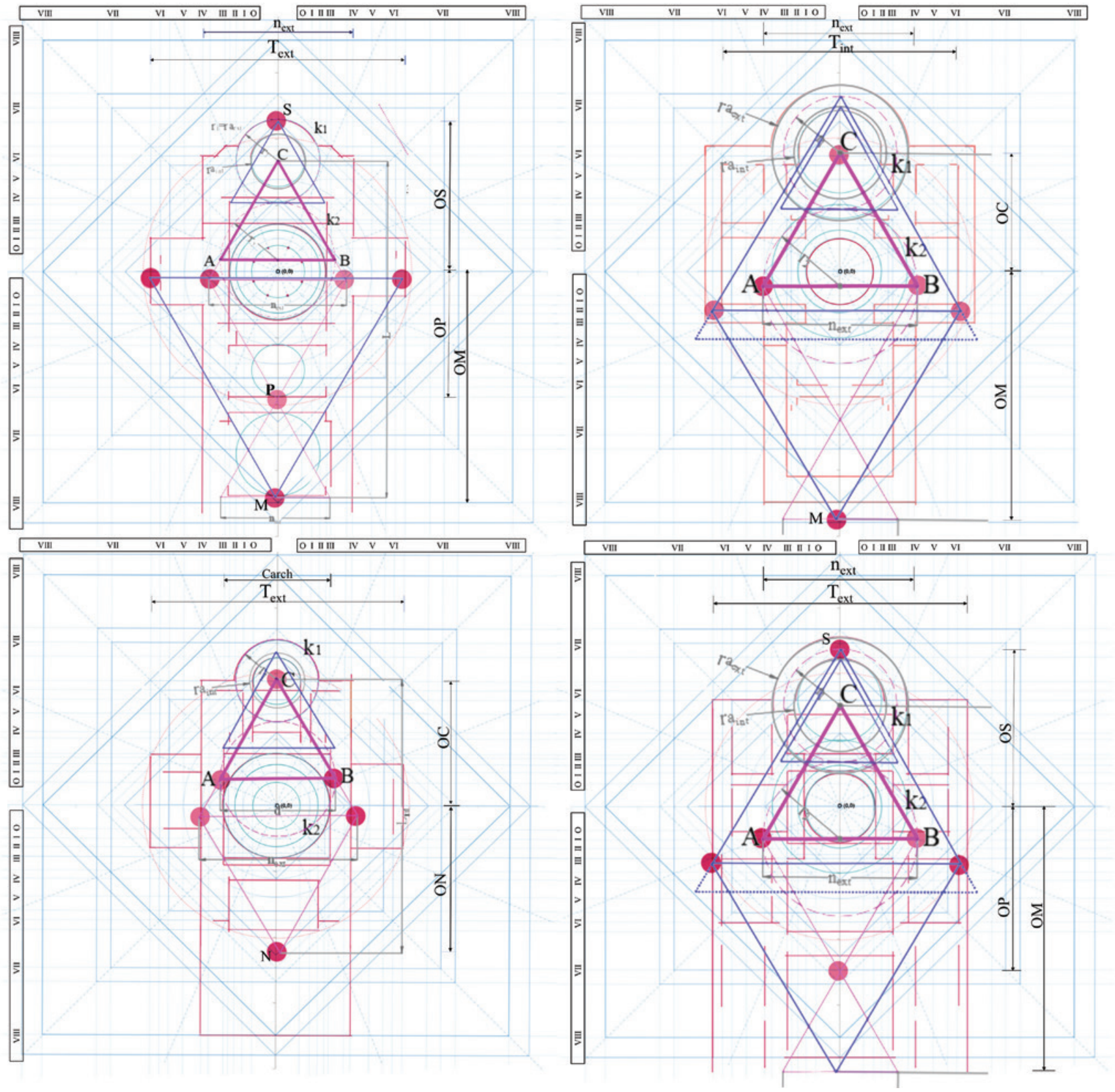


Fig. 6

A COMPARISON BETWEEN THE TWO GEOMETRIC CONCEPTS

As presented in Figure 6, and concerning the examination of a possible relation between Štambuk's canon with additional triangulation and the Octagon model, the two geometric concepts are overlapped by keeping the dispositions over the ground plans. Displaying both geometries over the ground plans of the churches enabled observing the common elements of the Octagon model and Štambuk's canon with additional triangulation. Namely, 5–7 points – vertexes of characteristic triangles lay over the elements of the Octagon model. Detailed elaboration of the correlations between the elements of the Octagon model and architectural spaces in the layouts, as well as Štambuk's canon with additional triangulation, is presented in Table 6.

Table 6 Relations of the Octagon model with the monuments' architectural spaces (model dimensions) and Štambuk's geometry

Octagon element	**Ø +**	Ø	**I**	**II**	**–**	**III**	**–**	**IV**	**–**	**VI**	**+**	**–**	**VII**	**VIII**	+
Value	$\frac{\sqrt{2}}{\alpha}$	$\sqrt{2}$	2	$\sqrt{2}\,\pi$	$\alpha\pi$	π	$\alpha\pi\sqrt{2}$	$\pi\sqrt{2}$	$\frac{\alpha\pi^2\sqrt{2}}{2}$	$\frac{\pi^2\sqrt{2}}{2\alpha}$	$\frac{\pi^2\sqrt{2}}{2\alpha}$	$\alpha\pi^2$	π^2	$\pi^2\sqrt{2}$	$\frac{\pi^2\sqrt{2}}{\alpha}$
Area		2	π	$0.5\pi^2$											
Geometry		square	circle	circle		square				square			square	square	
Studenica	apse		vestib.$_{Eext}$	apse$_{ext}$		c.core		nave Next		vestib Nint	vestib$_{Next}$ 1st aisle nave $_{Eext}$				nave Wint
Dimensions	192.2		267	294.6		336.3		518.3		801.1	917.8				
Štambuk	$r_{a\,int}$			k_1			n_{ext}				T_{ext}, OP	OS		OM	
St. Mary		apse$_{int}$ choir-Wint	choir $_{Eint}$	apse$_{ext}$ choir Wext		c.core choir$_{Wext}$		nave Next		nave$_{Next}$ choir$_{Nint}$ nave$_{Eint}$ narthex	nave$_{Eext}$			nave$_{Wext}$	
Dimensions		151.3		237.8		319.8		446.8			721.1			1398.9	
Štambuk				$r_{a\,int}$		c_{arch}					OC, T_{ext}	ON			
Arilje			dome choir $_{Wint}$	apse$_{int}$		choir$_{Wext}$ diakon$_{Wint}$ nave$_{Sint}$	apse$_{ext}$	n_{ext}	choir$_{N\text{-}int}$	nave$_{Next}$ 1st aisle Wint	nave$_{Eext}$ choir$_{ext}$			nave$_{Wext}$	
Dimensions		155		200			296	333.7			553			1019.5	
Štambuk							$r_{a\,ext}$	n_{ext}		OC	T_{int}				OM
Sopoćani			diakon$_{Wint}$ dome	apse$_{int}$		altar$_{int}$ nave$_{int}$			diakon Nint	chapel Wext	nave$_{Sext}$ choir$_{ext}$				nave Wint
Dimensions		214.9	240			310	397		579.9		751.1				1249
Štambuk				$r_{a\,int}$			$r_{a\,ext}$	n_{ext}			T_{ext}		OP		

CONCLUSION

Geometric analyses of the twelve ground plans of selected medieval churches in the Balkans are performed by applying the two diverse geometric methods. Eleven churches are from the Raška territory in medieval Serbia and belong to the same architectural style. Built contemporaneously with the early Raška churches towards the end of the twelfth century, the church of St. Mary on the island of Mljet in the southeastern Adriatic is used as an outlier monument.

The geometric method which employed Štambuk's canon with additional triangulation shows that triangular geometry is efficient in determining the general dimensions of the church layout, i.e. width of the nave, total width/length of the church and width of the altar. Simultaneously, the applied geometric shapes – equilateral triangles – are related to the characteristic points in the layout: the center and top of the apse, as well as the midpoint of the west entrance wall.

The results of this study indicate that most parts of the layouts of the four selected churches (in Studenica, Arilje, Sopoćani and St. Mary on the island of Mljet) can be analyzed by using the proposed geometry of the Octagon model. As expected, some of the key elements and points from Štambuk's canon appear within the Octagon model, such as the center of the apse, the edge of Štambuk's triangle.

Based on the results of this investigation, as well as the authors' previous research on this subject, it can be stated that these methods can be applied on a greater number of diverse churches regardless of their position in the larger Balkans region and regardless of their architectural style. This conclusion implies that the layouts of the churches could have been designed by applying similar geometric principles regardless of their execution and architectural style. It is also noteworthy that most of the churches share the same proportions, whether they are monumental in size, such as the churches in the monasteries Studenica, Žiča, Gradac and Sopoćani, or with modest dimensions, such as St. Achillius in Arilje.

Moreover, technological advances in computer aided design and contemporary software solutions aimed for parametric modeling, offer possibilities for advancing the process of redesigning the ruined churches based on here presented geometric methods. The research results show that only a few parameters, related to the given geometric schema, such as the width of the nave, and the total width, besides the wall thickness, determine the layout of the church in general terms. Simultaneously, the specific positions of geometric shapes: circles, triangles, and squares, within the layout are considered.

ACKNOWLEDGEMENTS

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ILLUSTRATIONS

1: Geometric schemas – representatives of the diverse architectural layouts for the Raška type of churches (according to Nenadović). Drawing: Magdalena Dragović.
Геометријске шеме – представници различитих основа цркава Рашког типа (према Ненадовићу). Цртеж: Магдалена Драговић.
2a: Štambuk's canon and additional triangulation: layouts of the churches of St. George – 1), St. Nicholas – 2), Sopoćani – 3) and St. Achillius – 4) with the corresponding geometry and schematic representations of the layouts. Drawings: Magdalena Dragović

Штамбуков канон са додатном триангулацијом – основе цркава: Св. Георгија – 1), Св. Николе – 2), Сопоћана – 3) и Св. Ахилија – 4) са одговарајућом геометријом и шемама основа. Цртежи: Магдалена Драговић.
2b: Štambuk's canon and additional triangulation: layouts of the churches of the monasteries in Gradac – 5), Studenica – 6), Žiča – 7) and Morača – 8) with the corresponding geometry and schematic representations of the layouts. Drawings: Magdalena Dragović.
Штамбуков канон са додатном триангулацијом – основе цркава манастира: Градац –5), Студеница – 6), Жича – 7) и Морача – 8) са одговарајућом геометријом и шемама основа. Цртежи: Магдалена Драговић.
2c: Štambuk's canon and additional triangulation: layouts of the churches of the monasteries in Mileševa – 9), Pridvorica – 10), Banjska – 11) and St. Mary – 12) with the corresponding geometry and schematic representations of the layouts. Drawings: Magdalena Dragović
Штамбуков канон са додатном триангулацијом – основе цркава у манастирима: Милешева – 9), Придворица – 10), Бањска – 11) и Св. Марија – 12) са одговарајућом геометријом и шемама основа. Цртежи: Магдалена Драговић.
3: The digitization of the ground plan of St. Achillius Church in Arilje. Drawing: Demetrius Savvides.
Дигитизација основе цркве Св. Ахилија у Ариљу. Цртеж: Деметриус Савидес
4: Octagon model set for the ground plans of: Studenica – a); Arilje – b); St. Mary – c); Sopoćani – d). Drawings: Demetrius Savvides.
Октагон модел постављен над основама цркава у Студеници – а), Ариљу – б), Св. Марији – ц) и Сопоћанима – д). Цртежи: Деметриус Савидес.
5: Correlation of the elements of Octagon model and the real dimensions of the layouts of: Studenica – a); Arilje – b); Sv. Marija – c); Sopoćani – d). Graphs: Demetrius Savvides.
Међузависност елемената Октагон модела и стварних димензија основа цркава: Студеница – а); Ариље – б); Св. Марија – ц); Сопоћани – д). Графикони: Деметриус Савидес
6: Mutual elements of the Octagon model and the Štambuk's canon with additional triangulation; Ground plans of: Studenica – a), St. Mary – b), Arilje – c) and Sopoćani – d). Drawings: Demetrius Savvides.
Заједнички елементи Октагон модела и Штамбуковог канона са додатном триангулацијом. Цртежи: Деметриус Савидес.

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ДВА ГЕОМЕТРИЈСКА КОНЦЕПТА ПРИМЕЊЕНА НА ПРОЈЕКТЕ СРЕДЊОВЕКОВНИХ ЦРКАВА НА БАЛКАНУ

Резиме: Овај рад представља комбиновану примену два геометријска концепта за анализу архитектонских основа одабраних средњовековних цркава на Балкану. Један концепт је геометријска конструкција, позната као Штамбуков канон, која је овде допуњена триангулацијом. Његова примена је већ потврђена у широком спектру касноантичких и средњовековних цркава на Балкану. Други концепт је Октагон модел, који се заснива на реконструисаним геометријским цртежима на камену пронађеним на рушевинама Октагона, оригиналном делу касноантичке Галеријеве палате у Солуну. Оба ова концепта се ослањају на геометријске облике и специфичне врсте грида, изведене комбиновањем основних облика – кругова са троугловима и квадратима. У првом приступу, две врсте низова троуглова, тј. геометријских образаца, примењују се на основе дванаест цркава. Анализа показује да ти обрасци одговарају за четири типа основа, које је претходно Ненадовић груписао. У другом приступу, заснованом на компјутерском моделовању и уклапању сложеног Октагон модела у основе четири цркве, максимално одступање је +/−3%. Иако су ово две различите почетне поставке за геометријску анализу, ова студија указује на јаку корелацију између њих. Наиме, постоји неколико заједничких елемената Штамбукове конструкције и Октагон модела, као што су центар апсиде, ивице Штамбуковог светог троугла и др.
Ово истраживање додатно указује на могућност да су споменици из ширег региона Балкана били пројектовани уз коришћење сличних основних геометријских принципа.
Кључне речи: геометријски концепт, касноримска архитектура, средњовековне цркве, Штамбуков канон, триангулатура, Октагон модел.